

Estimating the Consumption Function under the Permanent Income Hypothesis: The Case of Bangladesh

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Abstract:

This study aims to examine and estimate the consumption function in Bangladesh under Friedman's (1957) permanent income hypothesis (PIH) by adopting the adaptive expectations model. The study covers a data period of 40 years starting from FY 1982 to FY 2021 with annual observations in real terms. At first, a two-stage Engle-Granger cointegration test has been employed to estimate the elasticities of consumption both in the short and long run for Bangladesh along with an estimation of the adaptive expectations coefficient. The short-run MPC value is 0.6299. Consumption elasticity with respect to permanent income is 0.8397 provided the increase in income sustains in the long run. Given the values of consumption elasticity, it can be inferred that the consumers will realize 75% of their expectation in any period because the adaptive expectation coefficient has been estimated to be 0.7501. In the second approach, we regress changes in consumption as a function of changes in income and changes in one period lagged consumption since all these variables are found to be stationary at first difference. Both approaches lend support for consumption function formed in line with PIH and adaptive expectation hypothesis in Bangladesh.

Keywords: Consumption, Income, Permanent Income Hypothesis, PIH, MPC

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1.0 Introduction

Many researchers have tried to estimate the consumption pattern of different countries across the world to better understand the link between consumption and saving and then its relationship with overall economic behavior. Keynes' (1936) Absolute Income Hypothesis is such an early effort in this arena. Then similar types of seminal efforts were observed from the Kuznets Paradox (1946) by Kuznets, Life Cycle Hypothesis of Consumption (1954) by Franco Modigliani and his student Richard Brumberg, Permanent Income Hypothesis (1957) by Friedman, Random Walk Hypothesis (1978) by Hall, Campbell-Mankiw Consumption Function (1989)

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by Campbell and Mankiw, Relative Income Hypothesis (1989) by Duesenberry etc. Under the Absolute Income Hypothesis, Keynes described that the consumption of individuals depends on the current income and as current income increases, the consumption of the individuals also increases by a certain proportion of that current income. This idea faced several criticisms as it is observed that people do not instantly adjust their consumption pattern for a sudden positive or negative change in income. Then Friedman developed the PIH to explain the permanent consumption expenditure of individuals as a function of permanent income, not the current income. From then on, a number of studies are done to find the effectiveness of both models in different countries across the world. Similarly, this study aims to examine the consumption function of Bangladesh - a South Asian developing country under the Permanent Income Hypothesis over a period of 40 years ranging from 1982 to 2021. However, this study will follow a different approach of using real values to avoid the criticisms that were faced by many of the previous literature. Before directly delving into the analysis, this study presents an extensive literature review on both developed and developing countries including Bangladesh in the following section. And then in the methodology section, this study shows the econometric model extraction from the theoretical understanding of PIH along with data sources and the relevant statistical techniques that are required to analyze this hypothesis. Then in the results and discussion section, the study will analyze and interpret the statistical results and will try to conclude whether the results support the validity of PIH in Bangladesh or not.

2.0 Literature Review

According to Friedman (1957), the total income of an individual can be divided into two parts - permanent income and transitory income. Permanent income is expected to persist in the future whereas transitory income is random changes in the income level that will not persist in the future. Friedman stated that the permanent portion of income is the sole predictor of an individual's permanent consumption pattern. The permanent consumption of an individual will change due to only changes in the permanent income of that individual.

Since the development of PIH, several empirical studies have been done to test the consumption function with this PIH, e.g., DeJuan, Seater, and Wirjanto (1997) carried out a cross-country consumption function analysis including both developed and developing countries. The result shows that PIH holds true for developed countries but not for developing countries. Also, countries that maintain high-quality national income accounts find consistent results with the Permanent Income Hypothesis. They concluded that the stage of economic development and the quality of the national income account have significant effects on the effectiveness of PIH in the consumption function determination of a country. In a different study, they (2004) investigated the effectiveness of PIH in analyzing the consumption pattern of the

people within 48 states of the United States. Using the least square method they found that PIH predictions are credible. But they found no significant impact of income on consumption function in another study (2006) that they have conducted on West-German states with a data period of 1970-1997. However, Nawala (2010) used the Generalized Least Squares (GLS) method to find whether PIH holds for six developing countries in Africa. The study examined the effect of the change in fiscal policy and monetary policy on the aggregate expenditure of the countries. The study found that five of the six countries' consumption functions can be determined by the PIH. Khan and Mohammed (2011) used OLS Regression and instrumental variable method to study the PIH in the context of Pakistan, another developing country in Asia. They found that the Absolute Income Hypothesis helps to explain the consumption behavior of the people of Pakistan rather than the Permanent Income Hypothesis. Kundu and Mukhopadhyay (2011) used the Vector Error Correction model (VECM) to find whether PIH holds in the Sri Lankan economy, another developing country in the South Asian region. A proportional relationship was found between the permanent portion of consumption and the permanent portion of income. However, they found no relationship between permanent consumption and current income which means that the Permanent Income Hypothesis holds. Zarra-Nezhad, Saeidi, and Mansoury (2011) found the non-linear pattern of MPC for Iran as significant. They also found that it is current income that helps to explain private consumption rather than permanent income meaning the non-operability of PIH. They used empirical data from 1961 to 2009. Gomes (2012) carried out a similar analysis on the Brazilian economy using the ARIMA model with a data period of 1947-2010. The results found no importance of PIH in defining the consumption function. Elwakshi and Al-Qudah (2013) studied the consumption function of ten Arab countries for the period over 1985-2010 and found robust support for PIH. Altunç and Aydin (2014) tested the consumption function with PIH for eight member countries of the Organization of Islamic Cooperation (OIC). The study was conducted by using the Ordinary Least Squares method with a data period ranging from 1980 to 2010. They found that the results support PIH. Osei-Fosu, Butu, and Osei-Fosu (2014) analyzed the consumption function based on the economy of one of the developing countries of the African continent, Ghana. They used the adaptive expectation model to test the hypothesis and found that PIH holds for the people in Ghana.

In the case of Bangladesh, Islam (2010) found that income and one period lagged variable of consumption do not significantly explain consumption as their coefficients are not significant meaning that the Permanent Income Hypothesis does not hold. Khan, Anwar, Ahmed, and Kamal (2015) studied the consumption function considering both the hypotheses of Keynes and Friedman for the South Asian countries including Bangladesh. They found some inconsistent results for Bangladesh

under the Keynesian Absolute Income Hypothesis; but they found that PIH holds in the long run.

Most of the previous literature, as mentioned above, has addressed the relevance of PIH on consumption function by using nominal data rather than adjusting for inflationary effect. Moreover, this literature has incorporated the Ordinary Least Squares technique for the level data as the methodology with few exceptions. Both the above issues invite much criticism as these economic variables usually have unit root in most of the cases and also the real relationship gets masked due to the inflationary effect. That is why we have addressed these issues using a different methodology in this study and tried to find out whether PIH helps us to explain the consumption pattern of the people in Bangladesh.

3.0 Methodology

3.1 Theoretical Framework and Model Specification

As opposed to the simple Keynesian consumption function, Friedman's PIH; and Modigliani and Brumberg's life-cycle hypothesis proposed a different view of consumption by considering an individual's expected path of income or the time preference for consumption. They proposed that consumption was a function of average or projected income, or the worth of lifetime resources rather than a measured income as stated by Keynes.

The Permanent Income Hypothesis postulates that permanent income plays the largest role in influencing permanent consumption expenditures. According to the PIH, people make decisions about their permanent income using the adaptive expectations hypothesis as a framework. Friedman (1957) claims that the current consumption expenditure in a given period (C_t) consists of two components: the permanent consumption expenditure (C_t^P) and the temporary consumption expenditure (C_t^T) which can be written as:

$$C_t = C_t^P + C_t^T \quad (1)$$

In a similar way, current income in a particular period (Y_t) also consists of two components: permanent income (Y_t^P) and temporary income (Y_t^T) which can be written as:

$$Y_t = Y_t^P + Y_t^T \quad (2)$$

This fact leads to the assumption that permanent consumption expenditures are determined by permanent income. This functional relationship could be expressed in the following way while taking relevant factors into account:

$$C_t^P = \alpha + \beta Y_t^P \quad (3)$$

Here, α and β in equation (3) are to be estimated.

The study aims to calculate an estimate of a version of the consumption equation (3) or of Bangladesh's consumption function under PIH. Following the works of Katsouli (2006), Manitsaris (2006), Bonuedi (2012) and Altunc & Aydin (2014) we can convert the non-observable variable permanent income (Y_t^P) into actually measured variable using adaptive expectations model like the following:

$$Y_t^P - Y_{t-1}^P = \theta(Y_t - Y_{t-1}^P), 0 < \theta \leq 1 \quad (4)$$

Here, θ is the adaptive expectation coefficient. If we put the equation (3) in the equation (1), we will get the following equation:

$$C_t = \alpha + \beta Y_t^P + C_t^T \quad (5)$$

We can rewrite the equation (5) using the econometric terms like the following:

$$C_t = \alpha + \beta Y_t^P + u_t, \text{ where } u_t = \varepsilon_t + C_t^T \quad (6)$$

From equation (6), we can rewrite like the following:

$$Y_t^P = \frac{C_t}{\beta} - \frac{\alpha}{\beta} - \frac{u_t}{\beta} \quad (7)$$

Lagging equation (7) one period yields,

$$Y_{t-1}^P = \frac{C_{t-1}}{\beta} - \frac{\alpha}{\beta} - \frac{u_{t-1}}{\beta} \quad (8)$$

Inserting equation (7) and (8) in the equation (4) we can get:

$$C_t = \alpha\theta + \beta\theta Y_t + (1 - \theta)C_{t-1} + [u_t - (1 - \theta)u_{t-1}] \quad (9)$$

Here, $\alpha\theta$ = autonomous consumption; $\beta\theta$ = short run MPC; β = long run MPC and θ = the adaptive expectations adjustment coefficient which is (1-coefficient of C_{t-1}). Equation (9) is the consumption function under the PIH by using the adaptive expectations model. And this equation can be estimated as all the variables are in actual values. The derivation of equation (9) is given in the appendix.

3.2 Data

The study covers a data period of 40 years ranging from 1982 to 2021 with annual observations. The dependent variable, consumption, is defined by Households and NPISHs (Non-Profit Institutions Serving Households) Final Consumption Expenditure and the first independent variable, income, is defined by the GNDI (Gross National Disposable Income). The other independent variable is simply the one-period lagged variable of the dependent variable. These data series are secondary in nature and collected from the World Bank data repository. The study will use the GDP deflator to change the nominal values of the variables to real values to remove the inflationary effect in the data series. The study will also consider the logarithmic transformation of data to address the issues with regard to the unit root in time series.

3.3 Econometric Approach

As this study involves the analysis of time series data, the first task is to determine whether these data series have unit root or not. The choice of our statistical tools will also be determined by the findings of unit root conditions. So, for this, we will go for the Augmented Dickey-Fuller (ADF) test. If all the variables are stationary at level, then we will go for Ordinary Least Squares (OLS) Regression for the data series at level. But if the variables are found to be non-stationary at level and stationary at first difference, then we can follow two approaches. The first approach is using the two-stage Engle-Granger co-integration test where at the first stage the equation (9) can be estimated with OLS having at least MA(1). This is same as running ARIMA (0 0 1) on the equation of (9). In the second stage of the two-stage Engle-Granger cointegration test error terms can be found from the regression equation (9) with MA term for Bangladesh, thus the stationarity of the error terms series will be searched. If the error terms series from the equation (9) is integrated order of zero [$\varepsilon_t \sim I(0)$], then the error term will be regarded as white noise. In order to test whether PIH holds or not for Bangladesh, the second approach is to run a regression of changes in consumption ($C_t - C_{t-1}$) as a function of changes in Income ($Y_t - Y_{t-1}$) and changes in one period lagged consumption provided that C_t and Y_t are integrated order of one $\sim I(1)$ and error term of the regression is integrated order of zero.

4.0 Empirical Results and Discussion

4.1 Descriptive Statistics

The descriptive statistics in the following table have been determined to get a general idea about the variables under this study. We can understand the distribution pattern of the variables from these statistics, especially from skewness and kurtosis. And this understanding helps us in determining the appropriate statistical measures that we are required to undertake for a particular study.

Table-1 : Descriptive Statistics of Consumption, Income and One Period Lagged Variable of Consumption

Variables	Mean	Std. Dev.	Variance	Min	Max	Skewness	Kurtosis
$\ln C_t$	20.2519	.2280	.0520	20.0243	20.8611	1.3676	3.6744
$\ln Y_t$	20.5817	.2823	.0797	20.3274	21.2885	1.2992	3.3829
$\ln C_{t-1}$	20.1804	.2114	.0447	19.9636	20.7269	1.4067	3.8655

From Table-1, we can see that the standard deviations of all three variables are close to each other which means having similar fluctuations in the data structure. However, all three variables show high positive skewness and kurtosis at level which means a strong departure from normal distribution. In the subsequent sections, we will try to see whether these three variables with similarity in fluctuations, skewness and kurtosis can be modeled in such a way so that we can confidently estimate the effect on the dependent variable for any fluctuations in the independent variables.

4.2 Unit Root Test

The unit root test has been conducted by using the ADF test with the following two hypotheses:

$H_0 =$ *The variable has unit root or, the variable is not stationary*

$H_1 =$ *The variable does not have unit root or, the variable is stationary*

The results of the ADF test for unit root for each of the variables are:

Table-2 : ADF Test results for unit root

Variables	ADF Test Without Trend			ADF Test With Trend			Remarks
	Test Statistic	5% Critical Value	p-value	Test Statistic	5% Critical Value	p-value	
$\ln C_t$	1.753	-2.964	0.9983	-0.213	-3.548	0.9913	Non-stationary
$\Delta \ln C_t$	-4.985	-2.964	0.0000	-6.612	-3.548	0.0000	Stationary
$\ln Y_t$	1.946	-2.964	0.9986	-0.630	-3.548	0.9773	Non-stationary
$\Delta \ln Y_t$	-4.891	-2.964	0.0000	-6.454	-3.548	0.0000	Stationary
$\ln C_{t-1}$	0.766	-2.964	0.9911	-1.019	-3.548	0.9415	Non-stationary
$\Delta \ln C_{t-1}$	-5.387	-2.964	0.0000	-6.553	-3.548	0.0000	Stationary

From Table-2, we can see that all of the variables are non-stationary at level and stationary at first difference with 95% confidence level in both with and without trend situation of the ADF test. The optimal lag order for each of the variables at level was 1 and at first difference was 0. With this finding, we can employ the two-stage Engle-Granger cointegration test for estimating the consumption function under PIH by using the adaptive expectations model. Moreover, we can employ Ordinary Least Squares (OLS) regression of Δ consumption on Δ income and Δ in one period lagged consumption.

4.3 Results from Regression

The two-stage Engle-Granger cointegration test for equation (9) has been estimated where the first stage is to run the regression for consumption function with MA (1). The summary results from regression with MA (1) for $\ln C_t$ are presented in the following table:

Table-3 : Consumption Function of Bangladesh under PIH

	Coef.	OPG Std. Err.	Z	P> z	[95% Conf. Interval]	
$\ln Y_t$.6298662***	.1133848	5.56	0.000	.4076361	.8520964
$\ln C_{t-1}$.2498907*	.1433317	1.74	0.081	-.031034	.5308157
_cons	2.230307**	.9517994	2.34	0.019	.3648142	4.095799
MA (1)	.50735**	.240703	2.11	0.035	.0355807	.9791192
/sigma	.0266797	.0023564	11.32	0.000	.0220612	.0312982
Log likelihood	88.04787					
Number of Obs.	40					
Wald chi2 (3)	617.92					
Prob > chi2	0.0000					

Note: ***, ** & * represent significance at 1%, 5% & 10% respectively.

Table-3 shows the consumption elasticities with respect to income (short-run MPC) and one period lagged consumption for Bangladesh. Using these elasticities, we have calculated the adaptive expectations adjustment coefficient (θ) and consumption elasticity value based on permanent income (long run MPC) based on equation (4) and (9) respectively. These values are shown in the following table:

Table-4 : Estimated Parameters of Equation (9)

Consumption Elasticity with respect to actual income ($\beta\theta$) (Short run MPC)	Adaptive expectations adjustment coefficient (θ) (Equation 4)	Consumption Elasticity with respect to permanent income (β) (Long run MPC) (Equation 9)
0.6299	0.7501	0.8397

The short-run MPC ($\delta C_t / \delta Y_t$) is 0.6299 which means that a 1% increase in the current income will raise the current consumption by .6299% with other things remaining same. If this rise in income sustains, consumption elasticity with respect to permanent income (β , long run MPC) will be 0.8397. This implies that a 1% rise in

permanent income will cause the current consumption to rise by 0.8397%. Given short run MPC of 0.6299 and long run MPC of 0.8397, cautiously we can infer that consumers will realize around 75% of their expectations in any period as the θ is 0.7501.

At the second stage of the Engle-Granger two-stage cointegration test, error terms have been identified for the above regression model under equation (9) with MA(1) term for Bangladesh and have been tested for stationarity and were found to be integrated order of zero [$\varepsilon_t \sim I(0)$] as shown in the following Table-5. The test results are consistent with the consumption function which was formed under PIH and adaptive expectation hypothesis.

Table-5 : Unit Root Test for the Error term of the Regression Model in equation (9)

Variables	ADF Test Without Trend			ADF Test With Trend			Remarks
	Test Statistic	5% Critical Value	p-value	Test Statistic	5% Critical Value	p-value	
Residual (Error Term)	-5.302	-2.961	0.000	-5.981	-3.544	0.000	Stationary

Now, the results from OLS Regression of changes in current consumption ($\Delta \ln C_t$) on changes in income ($\Delta \ln Y_t$) and change in one period lagged consumption ($\Delta \ln C_{t-1}$) are presented in the following table:

Table-6 : OLS regression result for $\Delta \ln C_t$

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
$\Delta \ln Y_t$.9815534	.0696492	14.09	0.000	.8402982	1.122809
$\Delta \ln C_{t-1}$	-.1097623	.0619324	-1.77	0.085	-.2353669	.0158424
_cons	-.0030913	.0034742	-0.89	0.379	-.0101373	.0039548
Number of Obs.	39					
F (2, 36)	115.69					
Prob > F	0.0000					
R-squared	0.8654					
Adj R-squared	0.8579					

From Table-6, it can be seen that the independent variable, changes in income has a significant positive relationship with the dependent variable, changes in consumption. And the other independent variable changes in one period lagged consumption has a negative relationship with the dependent variable but this relationship is not significant at 5% level. Now to verify the viability of this long-run relationship we need to be sure about the stationarity in the error term of the above estimated model. We can estimate the residuals of the model by deducting the predicted values from the actual values of the changes in consumption. The results from the ADF Test of this residual are as follows:

Table-7 : Unit Root Test for the Error term of the above Regression Model

Variables	ADF Test Without Trend			ADF Test With Trend			Remarks
	Test Statistic	5% Critical Value	p-value	Test Statistic	5% Critical Value	p-value	
Residual (Error Term)	-7.969	-2.964	0.000	-8.087	-3.548	0.000	Stationary

From Table-7, we can see that the residual is stationary i. e., integrated order of zero. This means that the PIH holds in the long run for Bangladesh.

5.0 Conclusion

This study estimated the consumption function under PIH by adopting the adaptive expectation model for Bangladesh. While estimating the consumption function, the two-stage Engle-Granger cointegration test has been employed to estimate the short-run MPC and the consumption elasticity with respect to permanent income (long-run MPC). This cointegration test found that there exists a relationship between the nonstationary variables at level in the long run. The short-run MPC value is 0.6299. Consumption elasticity with respect to permanent income is 0.8397 provided the increase in income sustains in the long run. Given the values of consumption elasticity, it can be inferred that the consumers will realize 75% of their expectations in any period because the adaptive expectation coefficient is 0.7501. All the test results indicate that the PIH is valid for Bangladesh in the long run under the adaptive expectation model.

References

Altunc, O. F., & Aydin, C. (2014). An estimation of the consumption function under the permanent income hypothesis: the case of D-8 countries. *Journal of Economic Cooperation and Development*, 29-42.

- Bonuedi, I. (2012). *Estimating the Consumption Function under the Permanent Income Hypothesis: Evidence from Ghana*. Department of Economics, Kwame Nkrumah University of Science and Technology, Kumasi.
- Campbell, J. Y., & Mankiw, N. G. (1989). Consumption, income, and interest rates: Rinterpreting the time series evidence. . *NBER macroeconomics annual*, 185-216.
- DeJuan, J., Seater, J., & Wirjanto, T. (1997). A cross-country test of the permanent income hypothesis. *International Review of Applied Economics*, 451-468.
- DeJuan, J., Seater, J., & Wirjanto, T. (2004). A direct test on the permanent income hypothesis with an application to the US states. *Journal of Money, Credit, and Banking*, 1091-1103.
- DeJuan, J., Seater, J., & Wirjanto, T. (2006). Testing the permanent income hypothesis new evidence from West-German states. *Empirical Economics*, 613-629.
- Duesenberry, J. (1989). *Income, Savings, and the Theory of Consumption Behavior*. Harvard University Press.
- Elwakshi, M., & Al-Qudah, A. (2013). Consumption function under the permanent income hypothesis in selected Arab countries. *Proceedings of AICS-Social Sciences*, 166-170.
- Friedman, M. (1957). *A Theory of the Consumption Function*. Princeton: Princeton University Press.
- Gomes, F. (2012). A Direct test of the permanent income hypothesis: the brazilian case. *Brazilian Business Review*, 87-102.
- Hall, R. E. (1978). Stochastic implications of the life cycle-permanent income hypothesis: theory and evidence. *Journal of political economy*, 971-987.
- Islam, N. (2010). An Aggregate Consumption Function for Bangladesh: An Econometric Analysis. *Bangladesh Economic Association Shamoyiki-2010*.
- Katsouli, E. (2006). Testing the ‘Surprise’ Consumption Function: A Comparative Study between 15 European Union Member-states. *International Research Journal of European Union Economics and Finance*, 12-16.
- Keynes, J. (1936). *The General Theory Of Employment Interest rate and Money*.
- Khan, K., & Nishat, M. (2011). Permanent income hypothesis, myopia and liquidity constraints: A case study of Pakistan. *Pakistan Journal of Social Sciences (PJSS)*, 299-307.
- Khan, K., Anwar, S., Ahmed, M., & Kamal, M. (2015). Estimation of Consumption Function: The case of Bangladesh, India, Nepal, Pakistan, and Srilanka. *Pakistan Business Review*, 113-124.
- Kundu, A., & Mukhopadhyay, C. (2011). Consumption and income revisit of permanent income hypothesis with application to the economy of Srilanka. *International Coferance on Economics and Finance Research*. Singapore: IACSIT Press.
- Kuznets, S. (1946). *National Product Since 1869*. National Bureau of Economic Research.
- Manitsaris, A. (2006). Estimating the European Union Consumption Function under the Permanent Income Hypothesis. *International Research Journal of Finance and Economics*, 131-135.
- Modigliani, F., & Brumberg, R. (1954). Utility analysis and the consumption function: An interpretation of cross-section data. *Franco Modigliani*, 388-436.

Nwala, K. (2010). Does permanent income hypothesis hold for some selected countries? *African Journal of Business and Economic Research*, 27-34.

Osei-Fosu, A., Butu, M., & Osei-Fosu, A. (2014). Does Ghanaian's consumption function follow the permanent income hypothesis? The Cagan's adaptive expectation approach. *ADRRI Journal (Multidisciplinary)*, 133-148.

Zarra-Nezhad, M., Saeidi, S., & Mansoury, S. (2011). Estimation of Nonlinear Marginal Propensity to Consume in Iran. *European Journal of Economics, Finance, and Administrative Sciences*, 65-72.

Appendix

$$Y_t^P - Y_{t-1}^P = \theta(Y_t - Y_{t-1}^P), 0 < \theta \leq 1 \quad (4)$$

Here, θ is the adaptive expectation coefficient.

By inserting equation (3) in equation (1) we can get:

$$C_t = \alpha + \beta Y_t^P + C_t^T \quad (5)$$

By using econometric terms in equation (5), we can get:

$$C_t = \alpha + \beta Y_t^P + u_t, \text{ where } u_t = \varepsilon_t + C_t^T \quad (6)$$

$$\text{or, } \beta Y_t^P = C_t - \alpha - u_t$$

$$\text{or, } Y_t^P = \frac{C_t}{\beta} - \frac{\alpha}{\beta} - \frac{u_t}{\beta} \quad (7)$$

Lagging one-period yields:

$$Y_{t-1}^P = \frac{C_{t-1}}{\beta} - \frac{\alpha}{\beta} - \frac{u_{t-1}}{\beta} \quad (8)$$

Substituting equation (7) and (8) in the adaptive expectation equation (4) we can obtain:

$$\left(\frac{C_t}{\beta} - \frac{\alpha}{\beta} - \frac{u_t}{\beta} \right) - \left(\frac{C_{t-1}}{\beta} - \frac{\alpha}{\beta} - \frac{u_{t-1}}{\beta} \right) = \theta \left\{ Y_t - \left(\frac{C_{t-1}}{\beta} - \frac{\alpha}{\beta} - \frac{u_{t-1}}{\beta} \right) \right\}$$

$$\text{or, } \frac{C_t}{\beta} - \frac{C_{t-1}}{\beta} - \frac{u_t}{\beta} + \frac{u_{t-1}}{\beta} = \theta \left(Y_t - \frac{C_{t-1}}{\beta} + \frac{\alpha}{\beta} + \frac{u_{t-1}}{\beta} \right)$$

$$\text{or, } \frac{C_t}{\beta} - \frac{C_{t-1}}{\beta} - \frac{u_t}{\beta} + \frac{u_{t-1}}{\beta} = \theta Y_t - \frac{\theta C_{t-1}}{\beta} + \frac{\theta \alpha}{\beta} + \frac{\theta u_{t-1}}{\beta}$$

$$\text{or, } \frac{C_t}{\beta} = \theta Y_t + \frac{C_{t-1}}{\beta} - \frac{\theta C_{t-1}}{\beta} + \frac{u_t}{\beta} + \frac{\theta \alpha}{\beta} - \frac{u_{t-1}}{\beta} + \frac{\theta u_{t-1}}{\beta}$$

$$\text{or, } C_t = \beta \theta Y_t + C_{t-1} - \theta C_{t-1} + u_t + \theta \alpha - u_{t-1} + \theta u_{t-1}$$

$$\text{or, } C_t = \theta \alpha + \beta \theta Y_t + C_{t-1} - \theta C_{t-1} + u_t - u_{t-1} + \theta u_{t-1}$$

$$\text{or, } C_t = \alpha \theta + \beta \theta Y_t + (1 - \theta) C_{t-1} + [u_t - (1 - \theta) u_{t-1}] \quad (9)$$